Sheet (5)
DC Motors
Solution

Problem (1):

A 20hp, 250V shunt motor with R_a=0.22 Ω, R_f=170 Ω. At no-load and rated voltage, the speed is 1200 rpm and the armature current is 3 A. At full-load and rated voltage, the line current is 55A. What is the full-load speed?

Solution

At No-load

$$I_{f\,n.l} = \frac{V_t}{R_f} = \frac{250}{170} = 1.47\,A\ ,$$

$$N_{n.l} = 1200\ \ r.p.m$$

$$E_{n.l} = V_t - I_a R_a = 250 - (3)(0.22) = 249.34\,V$$

At Full-load

$$I_{f\,f.l} = \frac{V_t}{R_f} = \frac{250}{170} = 1.47\,A\ \ , N_{f.l} = \ ??$$

$$E_{f.l} = V_t - I_a R_a = 250 - (55)(0.22) = 238.22\,V$$

$$\frac{E_{n.l}}{E_{f.l}} = \frac{I_{f\,n.l} N_{n.l}}{I_{f\,f.l} N_{f.l}}$$

$$\frac{249.34}{238.22} = \frac{1200}{N_{f.l}}$$

$$N_{f.l} = 1146.5\ \ r.p.m$$

Problem (2):

A 230V shunt motor delivers 30hp at the shaft at 1120rpm. If the motor has an efficiency of 87% at this load, determine:

a) The total input power.
b) The line current.

Solution

(a) $\eta = \frac{P_{o/p}}{P_{i/p}}$

$$P_{i/p} = \frac{P_{o/p}}{\eta}$$

$$= \frac{30 * 746}{0.87}$$

$$= 25.72\ Kw$$

$$P_{i/p} = 25.72\ Kw$$

(b) $P_{i/p} = V_t I_l$

$$I_l = \frac{P_{i/p}}{V_t}$$

$$= \frac{25.72 * 10^3}{230}$$

$$= 111.84\ \ A$$

Good Luck ☺

Problem (3):

A separately excited motor runs at 1045rpm, with a constant field current, while taking an armature current of 50A at 120V. The armature resistance is 0.1 Ω if the load on the motor changes such that it now takes 95A at 120V, determine the motor speed at this load.

Solution

$$E = K_a \phi \omega = K \phi N$$

$$\frac{E_1}{E_2} = \frac{\phi_1 N_1}{\phi_2 N_2}$$

$\because$ *the field current is constant and the core is assumed unsaturated*

$$\therefore \ \phi_1 = \phi_2$$

$$\therefore \ \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$E = V_t - I_a R_a$$

$$E_1 = 120 - (50)(0.1) = 115\, V$$

$$E_2 = 120 - (95)(0.1) = 110.5\, V$$

$$\frac{115}{110.5} = \frac{1045}{N_2}$$

$$N_2 = 1004.1 \quad r.p.m$$

Good Luck ☺

Problem (4):

A separately excited DC motor has the following specifications:
Terminal voltage = 250 V, field voltage = 250 V, armature resistance = 0.03 Ω, field resistance = 250 Ω. Initially the motor was running at speed = 1103 rpm while supplied by the rated terminal voltage and the armature current = 120 A. While supplying constant torque, what is the speed of the motor if the terminal voltage is reduced to 200 V?

Solution

Torque is constant and V_f is not changed (the field flux will be constant)

$$T = K_a \phi I_a$$

$$T_1 = T_2$$

$$\phi_1 I_{a1} = \phi_2 I_{a2}$$

$$I_{a1} = I_{a2} = 120\ A$$

$$E = K_a \phi \omega = K \phi N$$

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$E_1 = V_{t1} - I_{a1} R_a = 250 - (120)(0.03) = 246.4\ V$$

$$E_2 = V_{t2} - I_{a2} R_a = 200 - (120)(0.03) = 196.4\ V$$

$$N_2 = \frac{E_2}{E_1} N_1$$

$$= \frac{196.4}{246.4} * 1103$$

$$= 879.5\ \ r.p.m$$

Good Luck ☺

Problem (5):

A 20 hp, 250 V DC shunt motor drives a load that requires a constant torque regardless the speed of operation. The armature resistance is 0.1 Ω. When this motor is running at full load, the armature current is 65 A at a speed of 1100 rpm. If the flux is reduced to 75% of its original value, find the armature current and the speed of the motor at this new condition?

Solution

Torque is constant

$$T_1 = T_2$$

$$\phi_1 I_{a1} = \phi_2 I_{a2}$$

$$\phi_1 I_{a1} = 0.75\phi_1 I_{a2}$$

$$I_{a1} = 0.75 I_{a2}$$

$$65 = 0.75\, I_{a2}$$

$$I_{a2} = 86.6\, A$$

$$E = K_a \phi \omega = K \phi N$$

$$\frac{E_1}{E_2} = \frac{\phi_1 N_1}{\phi_2 N_2} = \frac{\phi_1}{0.75\phi_1}\frac{N_1}{N_2} = \frac{1}{0.75}\frac{N_1}{N_2}$$

$$E = V_t - I_a R_a$$

$$E_1 = V_{t1} - I_{a1} R_a = 250 - (65)(0.1) = 243.5\, V$$

$$E_2 = V_{t2} - I_{a2} R_a = 250 - (86.6)(0.1) = 241.4\, V$$

$$\frac{243.5}{241.4} = \frac{1}{0.75}\frac{1100}{N_2}$$

$$N_2 = 1454 \quad r.p.m$$

Good Luck ☺

Problem (6):

The speed of 500 v shunt motor is to be raised from 700 rpm to 1000 rpm by field weakening the total torque remaining unchanged, the armature and the shunt field resistance are 0.8Ω ,750 Ω respectively and the supply current at lower speed is 12 A, calculate the additional shunt field resistance required.

Solution

Givens:

$V_t = 500\,V$ $\quad N_1 = 700 \ \ r.p.m$ $\quad N_2 = 1000 \ \ r.p.m$

$R_a = 0.8\,\Omega$ $\quad R_f = 750\,\Omega$ $\quad I_{l1} = 12A$

Required: $R_{f\ ext}$

case1: At lower speed

$$I_{f1} = \frac{V_t}{R_f} = \frac{500}{750} = 0.67\,A$$

$$I_{a1} = I_{l1} - I_{f1} = 12 - 0.67 = 11.33\,A$$

$$E_1 = V_t - I_{a1}R_a = 500 - (11.33)(0.8) = 490.93\,V$$

case2: At higher speed

$$T_1 = T_2$$

$$\phi_1 I_{a1} = \phi_2 I_{a2}$$

$$I_{f1} I_{a1} = I_{f2} I_{a2}$$

$$I_{a2} = \frac{7.59}{I_{f2}} \longrightarrow (1)$$

$$\frac{E_2}{E_1} = \frac{\phi_2 N_2}{\phi_1 N_1} = \frac{I_{f\,2} N_2}{I_{f\,1} N_1}$$

$$\frac{E_2}{490.93} = \frac{I_{f\,2}}{0.67}\frac{1000}{700}$$

$$E_2 = 1046.76\ I_{f\,2} \longrightarrow (2)$$

$$E_2 = V_t - I_{a2}R_a \quad \longrightarrow (3)$$

From (1) and (2) in (3) we get:

$$1046.76\ I_{f\,2} = 500 - \frac{7.59}{I_{f\,2}} * 0.8 \longrightarrow (4)$$

Rearranging (4)

$$I_{f\,2}{}^2 - 0.4776 I_{f\,2} + 5.8 * 10^3 = 0 \longrightarrow (5)$$

Solving (5) we get:

$I_{f\,2} = 0.465$ (Accepted near to $I_{f\,1}$) or $I_{f\,2}$=0.012 A (Rejected far from $I_{f\,1}$)

$$I_{f\,2} = \frac{V_t}{R_f + R_{f\,ext}}$$

$$R_{f\,ext} = \frac{V_t}{I_{f\,2}} - R_f$$

$$= \frac{500}{0.465} - 750$$

$$= 325\ \boldsymbol{\Omega}$$

Good Luck ☺

Problem (7):

A 500v, 10 HP, shunt motor has full load efficiency of 85% .for the same torque it is desired to reduce its speed by 30% by insertion of resistance in the armature circuit assuming that all the losses except copper losses vary directly with speed, calculate the value of the inserted resistance and the efficiency of the motor when running at the reduced speed, the resistance of the field and armature are 400 Ω and 0.25 Ω

Solution

Givens:

$V_t = 500\ V$ $\quad$ $P_{out} = 10\ HP,$ $\quad$ η=0.85 $\quad$ $,N_2 = 0.7\ N_1,$

$R_a = 0.25\Omega$ $\quad$ $R_f = 400\Omega$ $\quad$ $P_{loss\ except\ cu\ loss}\ \alpha\ N$

Required: $\quad$ $R_{a\ ext}$ $\quad$ η_2

Case (1)

$$\eta = \frac{P_o}{P_i} \quad \rightarrow \quad P_i = \frac{P_o}{\eta} = \frac{10 * 746}{0.85} = 8776.4\ Watt$$

$$P_i = V_t I_l \quad \rightarrow \quad I_l = \frac{P_i}{V_t} = \frac{8776.4}{500} = 17.55\ A$$

$$I_f = \frac{V_t}{R_f} = \frac{500}{400} = 1.25\ A$$

$$I_a = I_l - I_f = 17.55 - 1.25 = 16.3\ A$$

$$E_a = V_t - I_a R_a = 500 - (16.3)(0.25) = 495.9\ V$$

Case (2) *No change in the field circuit*

$$I_{f1} = I_{f2} = 1.25\ A$$

$$\frac{E_2}{E_1} = \frac{\phi_2 N_2}{\phi_1 N_1} = \frac{I_{f\,2} N_2}{I_{f\,1} N_1} = \frac{N_2}{N_1} \quad \rightarrow \quad E_2 = \frac{N_2}{N_1} E_1 = 0.7 * 495.9 = 347.1\ V$$

The same torque

$$T_1 = T_2$$

$$\phi_1 I_{a1} = \phi_2 I_{a2}$$

$$I_{f1} I_{a1} = I_{f2} I_{a2}$$

$$I_{a1} = I_{a2} = 16.3\ A$$

$$E_2 = V_t - I_{a2}(R_a + R_{a\ ext})$$

$$347.1 = 500 - (16.3)(0.25 + R_{a\ ext}) \qquad \rightarrow \qquad R_{a\ ext} = 9.127\ \Omega$$

$$\eta_2 = \frac{P_{i2} - P_{cu2} - P_{loss2}}{P_{i2}}$$

$$P_{i2} = V_t\left(I_{f2} + I_{a2}\right) = 500(16.3 + 1.25) = 8776.4\ w$$

$$P_{cu2} = {I_{a2}}^2(R_a + R_{a\ ext}) + {I_{f2}}^2 R_f = 16.3^2(0.25 + 9.127) + 1.25^2 * 400 = 3116.4\ w$$

$$P_{loss\ 2} = \frac{N_2}{N_1} P_{loss\ 1}$$

$$P_{loss\ 1} = P_{i1} - P_{cu1} - P_{o1}$$

$$P_{cu1} = {I_{a1}}^2(R_a) + {I_{f1}}^2 R_f = 16.3^2(0.25) + 1.25^2 * 400 = 691.4\ w$$

$$P_{loss\ 1} = P_{i1} - P_{cu1} - P_{o1} = 8776.47 - 691.42 - 7460 = 625\ w$$

$$P_{loss\ 2} = \frac{N_2}{N_1} P_{loss\ 1} = 0.7 * 625 = 437.5\ w$$

$$\eta_2 = \frac{P_{i2} - P_{cu2} - P_{loss2}}{P_{i2}} = 59.5\ \%$$

Note: Inserting an external armature resistance to control the speed considerable reduces the efficiency.

Good Luck ☺

Problem (8):

220 v, 7 Hp DC series motor coupled to a fan, the motor draws 25 A and runs at 300 rpm at terminal voltage of 220v and no external resistance, torque of the fan is proportional to the square of the speed, if R_a=0.6 Ω and R_f=0.4 Ω.

(**Neglect rotational losses**)

a) Determine the power supplied to the fan and torque developed by the motor.

b) The speed is to be reduced to 200 rpm by inserting external resistance, determine the value of this resistance and the power delivered to the fan.

Solution

Givens:

$V_t = 220\, V$ $\quad$ $P_{out\ rated} = 7\, HP$ $\quad$ $I_{a1} = 25A$ $\quad$ $N_1 = 300\, r.p.m$

$T\ \alpha\ N^2$ $\quad$ $R_a = 0.6\boldsymbol{\Omega},$ $\quad$ $R_f = 0.4\boldsymbol{\Omega}$

Required:

a) $P_{out} = ??\ ,\quad T_d = ??$

b) $R_{ext} = ??\quad and \quad P_{out} = ??\quad when\ N_2 = 200\, r.p.m$

Solution:

a) $E_a = V_t - I_a(R_a + R_f) = 220 - (25)(1) = 195\, V$

$$P_{out} \approx P_d = E_a * I_a = 195 * 25 = 4.875\, Kw$$

$$T_d = \frac{P_d}{\omega} = \frac{4.875\, Kw}{\frac{2\pi * 300}{60}} = 155.25\, N.m$$

b) $T\ \alpha\ N^2$

$$\frac{T_1}{T_2} = \left(\frac{N_1}{N_2}\right)^2$$

$$T_2 = \left(\frac{N_2}{N_1}\right)^2 T_1 = \left(\frac{200}{300}\right)^2 155.25 = 69\ N.m$$

Since for series DC motors $I_f = I_a$

$$\frac{E_2}{E_1} = \frac{\phi_2 N_2}{\phi_1 N_1} = \frac{I_{f2} N_2}{I_{f1} N_1} = \frac{I_{a2} N_2}{I_{a1} N_1}$$

$$\frac{E_2}{195} = \frac{I_{a\,2} * 200}{25 * 300}$$

$$E_2 = 5.2 I_{a2}$$

$$T_2 = \frac{E_2 * I_{a2}}{\omega_2}$$

$$69 = \frac{5.2 * {I_{a2}}^2}{\frac{2\pi * 200}{60}}$$

$$I_{a2} = 16.67\ A$$

$$E_2 = V_t - I_{a2}\left(R_a + R_f + R_{ext}\right)$$

$$5.2 * 16.67 = 220 - (16.67)(0.6 + 0.4 + R_{ext})$$

$$R_{ext} = 7\Omega$$

$$P_{out} \approx P_{d2} = I_{a2} E_{a2} = 5.2 * 16.6^2 = 1445\ w$$

Good Luck ☺